

In case you missed it, today's date is a rather fun one: 11/11/11. (It's also a date that Europeans and Americans write the same way. No fretting over "Should the day or month come first?" here.) Next year we'll have a 12/12/12, but then after that the "fun" dates will be few and far between. While we still have this fun date to enjoy — three prime numbers in a row! — let's revisit a lesson from the past about how to quickly break down larger numbers and determine whether or not they're prime.

Let's look at 2011. Is it a prime number? You could spend at least a few minutes trying to answer this question if you're not careful. But if you think strategically, it needn't take that long, and you can likely complete your "prime test" even within the two-minutes-per-question time allotment that the GMAT would give you for a question that, as so many do, requires your knowledge of prime numbers and divisibility. Here's how to get started:

1) When testing for primes, what you're really testing for is divisibility by prime numbers.

Let's just consider 13, which hopefully we all know is prime. Were we to test to be certain, though, we'd need to see if 13 is divisible by any numbers below it.

Well, 13 is not even, so it's not divisible by 2. And it's also not divisible by 3. We could test 4, but we don't need to — were 13 divisible by 4, it would already have been divisible by 2, because $4 = 2 \times 2$. Since we know that 13 is already not divisible by a factor of 4, we don't need to even test 4. Knowing this allows you to vastly minimize the number of integers you check when testing for prime.

2) Know the quick-test tricks for the most commonly-tested primes.

A number is only divisible by 2 if it's even. (The last digit is 0, 2, 4, 6, or 8.)

A number is only divisible by 3 if its digits sum to a multiple of 3 (e.g. 39 is divisible by 3 because $3+9 = 12$, a multiple of 3). NOTE: This trick also works for 9 (if the sum of the digits is a multiple of 9, the number is divisible by 9)

A number is only divisible by 5 if its last digit is 0 or 5

Outside of these first few, tricks do exist but unless you already know and regularly use them they may be more cumbersome than they're worth and they're certainly not as fast as the above; we'll explain later.

3) When testing for primes, you only have to look at divisibility by numbers up to the approximate square root of the number in question.

Consider a number like 71. Is 71 prime? To test, you'll again need to begin checking for divisibility by prime numbers:

2 – no, 71 is not even

3 – no, 71's digits do not add to a multiple of 3

5 – no, 71 does not end in 0 or 5

7 – no, 70 and 77 are multiples of 7, and 71 falls harmlessly in between

11 – similarly, 66 and 77 are consecutive multiples of 11, so 71, which falls between, is not

Now, this could go on for a while, and the further we get from single-digit integers the harder it becomes to test for divisibility. How far need we go? If you look at a perfect square just above 71, so let's use 81, you can see that:

$$81 = 9 \times 9$$

If our goal is to stay below that threshold of 81, we would then need to reduce one of the 9s that are multiplied to get 81. We certainly can't increase or we'd go above. So, we can prove that in order to have 9 or anything greater multiplied to get to 71, we'd need to have something less than 9 as the other partner in multiplication. And we've already tested all of the prime numbers below 9, so we're done. Once 7 wasn't a factor of 71, since 8 and 9 are neither prime, we could prove that 71 is prime. The only way that anything greater than 9 would be a factor is if a number less than 9 were already a factor; that isn't the case, so 71 is prime.

So...that brings us to 2011, which is a considerably larger number than 71 and likely larger than any number the GMAT would provide as a "prime candidate." But using what we already know, we can make this a fairly efficient process nonetheless. Is 2011 divisible by:

2 – no, it's not even

3 – no, its digits sum to 4

5 – no, it doesn't end in 0 or 5

7 – here's where we may need to get creative. We could perform the long division, but we could also do as we did above with 7 and 11 and find "easy to calculate" multiples of 7 and work from there. We should know that 2100 is a multiple of 7 (21 is, so multiplying it by 100 keeps it a multiple of 7). And $2100 - 70 = 2030$, so we know that's another multiple of 7. Then subtract another multiple of 7 (say, 21): $2030 - 21 = 2009$. So we know that if 2009 is a multiple of 7, and 2011 is only two spots away, it's not divisible by 7. We can play this game with all other numbers should we want to avoid long division.

11 – playing that same game: 2200 is a multiple of 11. Subtract another multiple: $2200 - 110 = 2090$. Subtract another to get close to 2011: $2090 - 77 = 2013$. So we can prove that 2011 is not a multiple of 11.

13 – 2600 is a multiple of 13. Subtract 390, another multiple (it's easiest to find multiples that end in 0...you just take a multiple you know and tack on a 0 to have that $\times 10$), and you have 2210. Subtract 130 to get to 2080. Subtract 39 to get 2041, then 26 to get 2015 and you can recognize that 2011 is not a multiple of 13.

17 – 1700 is a multiple of 17; add 340, another multiple, to get closer to 2011 and you have 2040. Subtract 17 and that's 2023, and subtract 17 again to get that range ($2023 - 17 = 2006$) and recognize that 17 is not a factor of 2011.

19 – 1900 is a multiple of 19. Add 95 (190 divided by 2) to get 1995. Add 19 to get 2014 and we can prove that 19 is not a factor of 2011.

23 – 2300 is a multiple of 23. Subtract 230 to get 2070. Subtract 69 to get 2001, add 23 to bracket the range at 2024 and you can tell that 23 is not a factor of 2011.

29 – 2900 is a multiple of 29. Subtract 290 to get 2610; do it again to get 2320, and one more time to get to 2030. Subtract 29 again and you'll find that 2001 and 2030 are factors of 29, so 2011 is not.

31 – 3100 is a multiple of 31. Subtract 930 to get 2170. Subtract 93 to get 2077. Subtract 62 to get 2015, and we can prove that 2011 is not divisible by 31.

37 – 3700 is a multiple of 37, but far too large, so divide by 2 to get closer to the intended range: 1850. Add 185 to get 2035. Subtract 37 to establish the range around 2011 and you have 1998, so 37 is not a factor of 2011.

Let's top and see where we are: We're looking for 2011, which is significantly less than 2500, or 50×50 , so we don't have to try any primes above 50...we're almost there!

41: 4100 is a multiple of 41, and if we cut that in half we're in the range at 2050. Subtract 41 to get 2009 and you can realize that 41 won't be a factor of 2011.

43: 4300 is a multiple of 43, and if we cut that in half we're close at 2150. Subtract 86 to get 2064, and then subtract 43 to get 2021. 43 won't be a factor of 2011.

47: 4700 is a multiple of 47. Cut it in half: 2350. Subtract 235 to get closer and you're at 2115. Subtract 94, another multiple, to get to 2021, and it's clear that 2011 is not divisible by 47, either.

Don't stress too much... 2011 is probably a bigger number than you will have to test for primes on the GMAT, but notice how the job gets easier with the larger numbers because the interval between multiples of larger numbers is so great. If you keep in mind that you only need to test primes; that you only have to test up to the square root of the next highest square, and that you can play the multiples game instead of performing long division, prime number testing can be done much more efficiently than you might think!